Mechanically triggered transformations of phononic band gaps in periodic elastomeric structures

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We present a full acoustic band structure calculation for periodic elastomeric solids at different levels of deformation. We demonstrate the ability to use deformation to transform phononic band gaps. Periodic elastomeric structures are subjected to axial compression and are found to undergo a transformation in their patterned structure upon reaching a critical value of applied load. During the initial linear regime of the nominal stress-strain behavior, the band gaps evolve in an affine and marginal manner. Upon reaching the critical load, the pattern transformation is found to strongly affect the in-plane phononic band gaps, resulting in the closure of existing band gaps and in the opening of new ones. The elastomeric nature of the material makes the transformation in both structural pattern and phononic band gap a reversible and repeatable process, creating a phononic band gap switch.

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Phononic crystals are periodic elastic structures which exhibit a range in frequency where elastic wave propagation is barred. The ability to design structures with such phononic band gaps (PBGs) has been of growing interest in recent years due to their potential as sound filters, acoustic mirrors, acoustic wave guides, and vibration isolators and in transducer design. Typical structures take the form of two-dimensional (2D) or 3D arrays of inclusions of one (or more) material(s) in a matrix with contrasting properties. The position and width of the PBGs can be tailored by the selection of (i) constituent materials with contrasting densities and contrasting speeds of sound, (ii) lattice topology (for example, square versus rectangular versus oblique arrays), (iii) lattice spacing, and (iv) volume fraction of inclusions. Many solid-solid, solid-fluid, and solid-air structures have been pursued through experiments and/or simulations. Studies have focused on determining the properties of particular material systems,[1–9] on developing modeling and/or optimization strategies for designing phononic crystal structures,[1,10–13] and on designing materials which exhibit both phononic and photonic band gaps.[14]

Recent investigations focusing on tunable phononic band gap systems have shown that the properties of phononic crystals can be modified by (i) using the piezoelectric effect which altered out-of-plane modes,[15] (ii) through direct physical rotation of elements in a 2D periodic system of rods hosted in air,[16] and (iii) through direct physical changing of the positioning and dimensions of the periodic geometry.[17,18] However, to our knowledge, the use of deformation to tune and transform the band structure of periodic elastomeric solids has never been considered. Furthermore, analysis of these physics requires more sophisticated calculations which account for the effects of the nonlinear deformation (including nonlinear material behavior, nonlinear geometry effects which accompany finite deformations, and inhomogeneous stress fields which develop with deformation) on the propagation of elastic waves.

An elastomer can reversibly undergo small to large strain deformations and can be exploited within both its linear and nonlinear regimes of elastic deformation. The propagation of elastic waves through the material is affected by the level of deformation and such an effect has to be taken into account in the calculations. To this purpose we developed a numerical technique based on the finite-element method for the analysis of wave propagation through elastic solids subjected to finite nonlinear deformation. Recently, 2D periodic elastomeric structures have been shown to undergo dramatic mechanically triggered transformations in their periodic pattern.[19,20] For example, during axial compression, a square array of circular holes in an elastomeric matrix was found to suddenly transform, upon reaching a critical applied stress, to a periodic pattern of alternating, mutually orthogonal, ellipses (Fig. 1, top), while an oblique array was found to transform into one of sheared voids where the shear direction alternates back and forth from row to row (Fig. 1, bottom). Upon removal of the stress, the initial periodic structure is recovered, giving a reversible and repeatable process. In this work, we demonstrate through modeling that these mechanically triggered pattern transformations can be further exploited to transform the character of the PBG structure of the

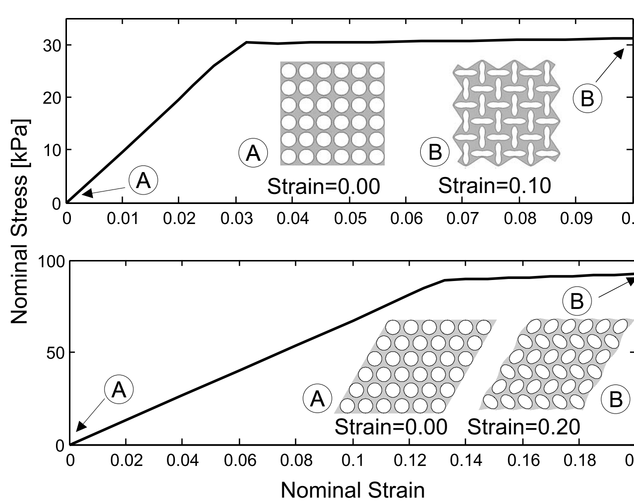

FIG. 1. Nominal stress vs nominal strain curves during axial compression in the vertical direction for the square (top) and hexagonal (bottom) arrays of circular holes. The departure from linearity is the result of an elastic buckling in the microstructure that triggers a pattern transformation.

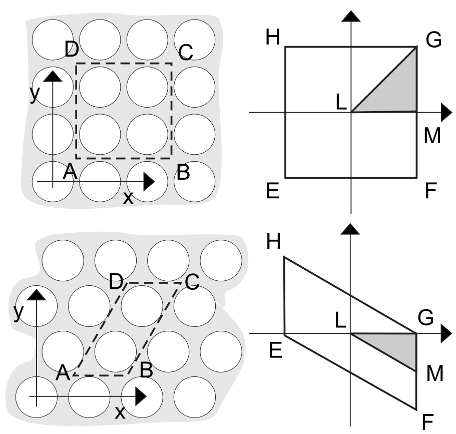

FIG. 2. Square (top) and oblique (bottom) infinite array of circular voids: cell in the direct lattice (left) and its corresponding in the reciprocal lattice (right). The perimeter of the gray area illustrates the contour along which the eigenfrequencies are plotted.

material, opening up different avenues of tailoring and control in acoustic design.

The mechanically triggered transformative character of the PBGs in periodic elastomeric structures is examined by studying two representative 2D infinitely periodic structures: a square array of circular holes of radius $r=4.335$ mm in an elastomeric matrix with center-to-center spacing $a_x=a_y=9.97$ mm, so that the initial void volume fraction is $f_0=0.59$ (Fig. 2, top) and an oblique array of circular holes of radius $r=4.335$ mm with center-to-center spacing $a_y=9.47$ mm vertically and $a_x=10.97$ mm horizontally (Fig. 2, bottom) with an initial void volume fraction $f_0=0.57$. The pattern transformation behavior of these arrays was studied in Ref. 19. The stress-strain behavior of the elastomeric matrix material was found to be captured using a nearly incompressible nonlinear second-order I_1 (first invariant of the strain) Rivlin hyperelastic model as in Ref. 19. The initial shear modulus is 1.08 MPa and bulk modulus is 2 GPa; the elastomer density is 1050 $\mathrm{kg/m^3}$ so that the transverse and longitudinal speeds of sound for the undeformed material are $c_{t0}=32.2$ m/s and $c_{l0}=1312$ m/s. When the periodic elastomeric structure is subjected to axial compression, a dramatic pattern transformation is observed to occur.[19,20] Each array exhibits an initial linear elastic behavior with a sudden departure from linearity to a plateau stress (Fig. 1). The departure from linearity is a result of a sudden transformation in the periodic pattern as shown in the inset of Fig. 1 as well in Figs. 3 and 5 (left). Using a Bloch-wave analysis, it has been shown in Ref. 19 that the pattern transformations for the infinite periodic structures are a result of an elastic instability in the cell microstructure. The bifurcation introduces a periodic cell larger than the primitive cell of the lattice. Thus, in correspondence with the periodicity of the transformed patterns, representative volume elements (RVEs) consisting of 2×2 and 1×2 primitive cells are considered for the square and oblique arrays of circular holes, respectively, in our simulations of both the nonlinear deformation and the PBG structure (Fig. 2).

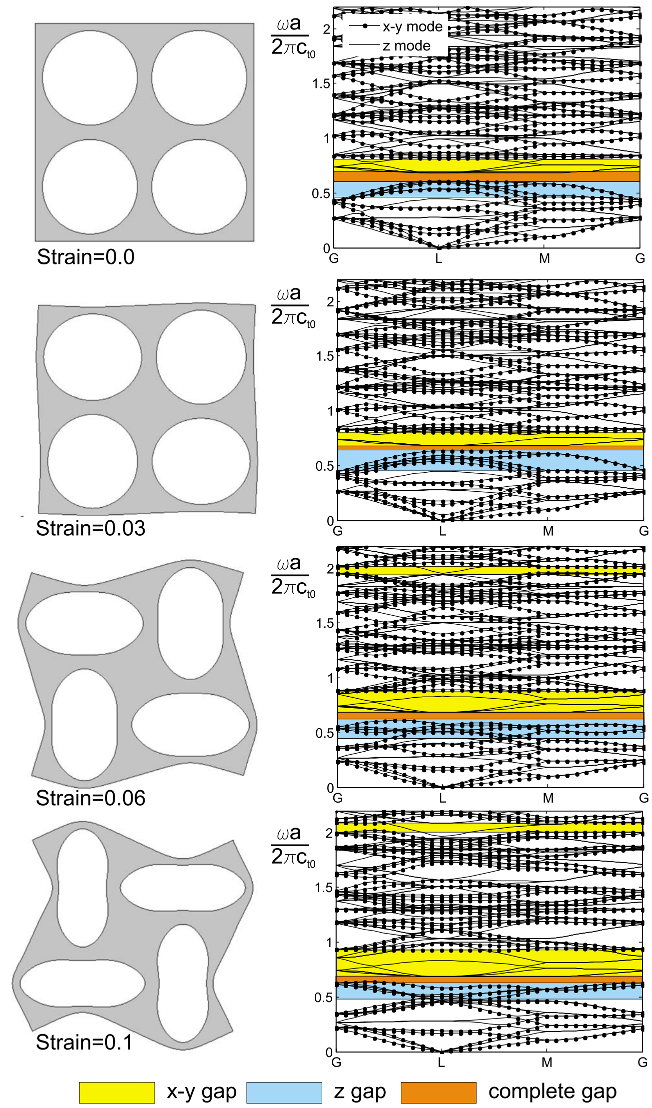

FIG. 3. (Color online) Phononic band gap structure for the square array of circular holes with $r=4.335$ mm and $a_x=a_y=9.97$ mm ($f_0=0.59$) at different levels of macroscopic nominal strain in the elastomer matrix ($c_{t0}=32.2$ m/s). In-plane (labeled x-y) and out-of-plane (labeled z) modes are reported together with the x-y, z, and complete band gaps. The points G, L, and M are defined in Fig. 2.

The nonlinear finite-element code ABAQUS was used to deform the structures as well as to obtain the dispersion diagrams. A 3D mesh of each RVE was constructed using 15-node hybrid wedge elements (only one layer of elements is used in the z direction). The RVE is subjected to macroscopic axial compression. The deformation is applied to the surface of the RVE through a series of constraint equations which provide general periodic boundary conditions and respect the infinite periodicity of the structure.

The propagation of elastic waves through each structure is analyzed at different levels of macroscopic strain. The finite-element method is also used to compute the band structure (following, for example, Refs. 11 and 12). This necessitates conducting a Bloch-wave analysis within the finite-element framework. In order to work with the complex-valued dis-

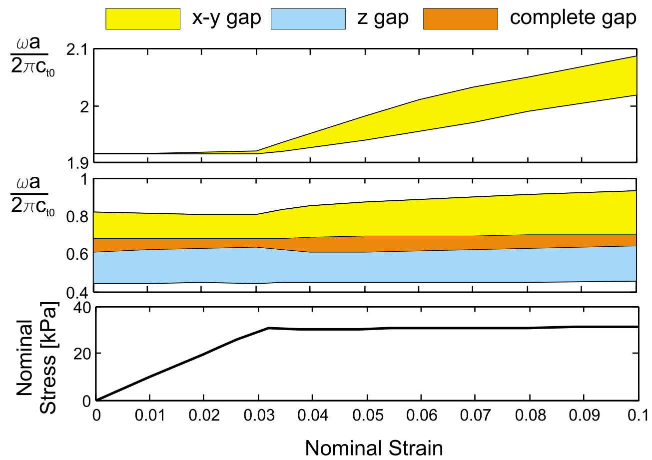

FIG. 4. (Color online) Phononic band gap (top and center) and nominal stress (bottom) vs nominal strain for the square array of circular holes.

placements of the Bloch-wave calculation within the confines of a commercial code, all fields are split into real and imaginary parts.[12] In this way the equilibrium equations split into two sets of uncoupled equations for the real and imaginary parts. Thus the problem is solved using two identical finite-element meshes for the RVE, one for the real part and one for the imaginary part, and coupling them by Bloch-type displacement boundary conditions. In this way eigenfrequencies ω can be computed for any wave vector $\mathbf{k}_0$. Here the wave propagation is limited to the x-y plane perpendicular to the holes ($k_{0z}=0$), so that a decoupling between the out-of plane (z) and in-plane (x-y) wave polarizations is obtained.

The band diagrams for the case of the square array of circular holes are provided at different levels of macroscopic nominal strain in Fig. 3. Both in-plane (x-y) and out-of-plane (z) modes are shown together with the evolving structure. The transformation of the band gaps with deformation is reported in Fig. 4. In the undeformed configuration, the periodic structure possesses an in-plane (x-y) phononic band gap for normalized frequencies of $\tilde{\omega}=\omega a/(2\pi c_{t0})=0.61-0.82$ [with $a=(a_x+a_y)/2$] and an intersecting out-of-plane (z) gap for $\tilde{\omega}=0.45-0.68$, yielding a complete phononic band gap for $\tilde{\omega}=0.61-0.68$ (Fig. 3, top). During the initial linear elastic response of the periodic structure, the circular holes are observed to undergo a gradual and homogeneous compression (Fig. 3, top). At this stage the band gaps are affected marginally by the deformation, evolving in an affine and monotonic manner (Fig. 4). This relatively affinelike behavior is replaced by a transformation to a pattern of alternating, mutually orthogonal ellipses above a nominal strain of 0.032. The in-plane (x-y) modes undergo a transformation as well, while the out-of-plane (z) modes are observed to be only marginally affected by the pattern transformation. A new in-plane (x-y) band gap is opened at $\tilde{\omega}=2$, and the preexisting gap now begins to widen.

The band diagrams for the case of the oblique array of circular holes are shown in Fig. 5 for both the in-plane (x-y) and out-of-plane (z) modes at different levels of macroscopic nominal strain. The transformation of the band gaps with deformation is reported in Fig. 6. The undeformed configuration of the periodic structure possesses three separate out-of-plane (z) band gaps for $\tilde{\omega}=[0.92-0.94, 1.6-1.8, 2.0-2.16]$.

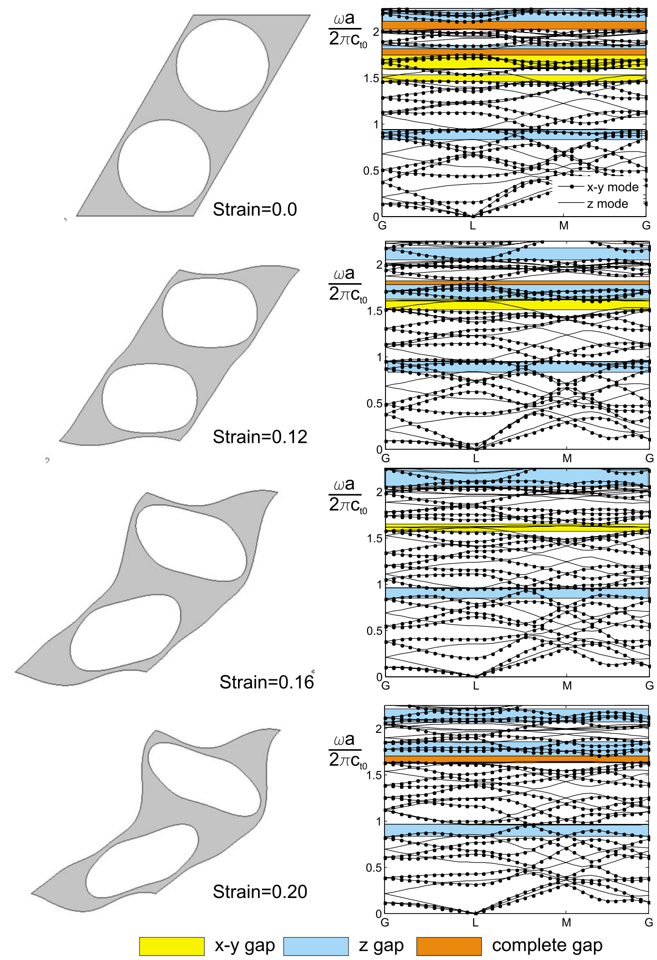

FIG. 5. (Color online) Phononic band gap structure for the oblique array of circular holes with $r=4.335$ mm, $a_x=10.97$ mm, and $a_y=9.47$ mm ($f_0=0.57$) at different levels of macroscopic nominal strain in the elastomer matrix ($c_{t0}=32.2$ m/s). In-plane (labeled x-y) and out-of-plane (labeled z) modes are reported together with the x-y, z, and complete band gaps. The points G, L, and M are defined in Fig. 2.

As in the case for the square array, these band gaps are only marginally affected by the pattern transformation. The undeformed structure exhibits three separate in-plane (x-y) band gaps for $\tilde{\omega}=[1.45-1.52, 1.75-1.85, 2.05-2.12]$. The width of the lowest-frequency gap is not strongly affected by the deformation and transformation, but at a strain of 0.13 it intersects the z mode, yielding a complete band gap. The width of the second and third gaps is observed to reduce progressively with increasing deformation until a strain of 0.125 whereupon the pattern transformation yields their complete closure.

We have uncovered the ability to transform phononic band gaps in elastomeric periodic solids using the simple application of an axial load. Periodic elastomeric structures have been shown to be characterized by an initial affinelike

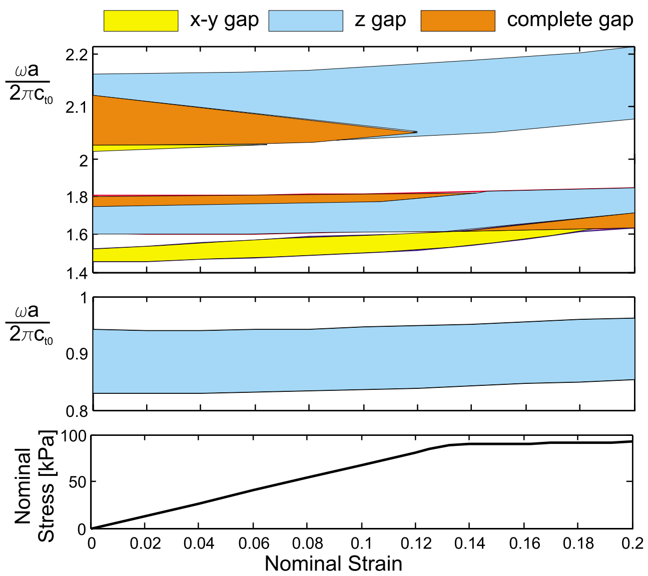

FIG. 6. (Color online) Phononic band gap (top and center) and nominal stress (bottom) vs nominal strain for the oblique array of circular holes.

deformation, followed by a homogeneous pattern transformation upon reaching a critical value of applied load.[19,20] In the present work it has been shown that the phononic band structure evolves in a monotonic manner during the linear region of nominal stress-strain behavior when the deformation of the inherent structure pattern is relatively affine. When the periodic pattern transforms to a new pattern upon reaching the critical load, the evolution in the phononic band gap also changes in a nonaffine manner. For the particular geometry and properties studied here, the band gaps exhibited by the materials are in the audible range. The location and presence of the gaps as well as their transformation can be further tuned by varying the geometric properties of the periodic structures (e.g., the initial pattern and lattice spacing) as well as selecting different material properties (e.g., matrix material stiffness, density, and inclusion properties). The transformations can be further manipulated applying different types of loading (e.g., biaxial loading or shear loading), using different materials (e.g., anisotropic materials, dielectric or viscoelastic elastomers) and would also extend to three-dimensional periodic structures. In these ways the band gaps and their transformations can be tuned not only for the audible range, but also for other frequency domains of interest. Furthermore, the mechanically triggered pattern transformation phenomena can be utilized in photonic applications using appropriate materials and pattern length scales.

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